Growth and Innovation in the Presence of Knowledge and R&D Accumulation Dynamics

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This article develops a model of growth and innovation in which accumulation dynamics of knowledge and R&D are explicitly considered. The model is based on a more general knowledge production framework than commonly used in Endogenous Growth Theory and R&D productivity literatures, reconciling as special cases disparate analytical frameworks and functional forms. The modeling approach reveals the structure of R&D elasticity estimation biases that can result from failure to take into account the accumulation dynamics of knowledge and R&D. These findings provide guideposts for empirical studies on R&D productivity and innovation based on the knowledge production function framework. Finally, the model of knowledge sector dynamics highlights the role of physical capital in the creation of innovations and establishes the theoretical possibility of long-run idea-driven growth without the razor-edge assumption of Romer (1990) and in the absence of growth in R&D employment stipulated by Jones (1995).
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I. Introduction

Scholars from different academic disciplines, and working with different methodologies, argue that accumulation of technological knowledge is a key driver of economic growth. Historical accounts of the economic development of nations observe how mastery of new technologies had accompanied industrial growth (Gerschenkron, 1962). Knowledge also occupies a central role in growth theory, where it figures as a key input, alongside capital and labor, in models of aggregate production (Solow, 1956, 1957; Romer, 1990). The consensus on the centrality of knowledge accumulation to economic growth extends to many sub-disciplines of economics, including production economics, economics of management, and

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economics of innovation. Because knowledge is widely believed to be an important ingredient in economic growth, the process of knowledge production and accumulation deserves careful attention.

In this paper we present a model of knowledge dynamics designed to capture the process of knowledge creation and accumulation. The model extends the knowledge production function encountered in models from Endogenous Growth Theory (EGT) and incorporates features of knowledge capital accumulation appearing in the literature on R&D and productivity. In the proposed model, we make a distinction between R&D and knowledge, which are closely related and frequently conflated concepts in the literature. The model includes two stocks: a knowledge stock consisting of the sum total of disembodied technologically relevant ideas, and an R&D stock, representing accumulated embodied research effort. R&D stock contains labor and physical capital components, allowing for a role of physical capital in the creation of knowledge. Maintaining separate stocks allows us to capture the separate but interconnected flow and accumulation dynamics of knowledge and R&D.

This analytical exercise has implications for balanced growth and the measurement of returns to R&D. Our model of knowledge dynamics reveals the structure of estimation biases that can result from exclusion of physical capital and failure to take into account accumulation dynamics. Lastly, the model suggests the possibility of idea-driven growth without the razor-edge assumption of Romer (1990) and in the absence of growth in R&D employment stipulated by Jones (1995).

Section IV introduces the R&D-based knowledge production function which is a modification of the knowledge production function from EGT. Section V sets up the building blocks of a model of knowledge dynamics. Section VI develops the full model of knowledge and R&D accumulation dynamics, while Section VII discusses the model's comparative statics. Sections VIII and IX identify the implications of the model for measurement and econometric estimation of R&D elasticity of innovation. Section X discusses interesting implications of the model from the point of view of economic growth. But first, in Sections II and III, we try to set aside a few common misconceptions.

II. Reflections on Models of Knowledge Production

The *production function* is one of the central analytical tools in economics. By relating output to its factor inputs it describes the production process and links to other notions from production theory, such as efficiency and productivity. Production functions used in contemporary economics trace their lineage to the 1928 work of Charles Cobb and Paul Douglas (Berndt and Christensen, 1973), although the idea of expressing production as a mathematical function that relates output to a set of inputs is older, dating at least as far back as 1894, when philosopher Philip Wicksteed published his essay on distribution (Wicksteed, 1894), and possibly to earlier work of Johann von Thünen (Mishra, 2007). The *knowledge production function* (KPF) framework represents one important methodological approach to the study of innovation and technical change—an alternative to qualitative and historical studies.[1]

While in a generic production function an index of outputs is related to measures of factor inputs, in a knowledge production function an index of innovation is related to factors determining the intensity of innovative activity. The index of innovation is either incorporated into a broader production function framework which includes output, or is used standalone. The latter is more frequently the case in applied and theoretical research that is concerned primarily with innovation itself and only to a lesser extent with the role of innovation in a broader production system.

The knowledge production function approach has been applied to assess the impact of R&D on output and total factor productivity (Griliches, 1988; Verspagen, 1995; Abdih and Joutz, 2006), to estimate the rate of return to R&D (Bernstein, 1989; Jones and Williams, 1998), to understand factors determining the intensity of innovative activity across industries and at various spatial scales (Porter and Stern, 2000; Mohnen, Mairesse and Dagenais, 2006), and to measure knowledge spillovers (Jaffe, 1986; Griliches, 1992; Coe and Helpman, 1995; Audretsch and Feldman, 1996).[2]

Knowledge production in the R&D productivity literature

Knowledge production functions come standard with the literature on productivity, an early discussion of which can be found in Griliches (1979). The departure point for these studies is the aggregate production function:

$$Y = F(A, K, L) \tag{1}$$

in which a measure of output Y is related to inputs, where K and L represent capital and labor, respectively, and A stands for the level of technological knowledge. The literature posits a relationship between the level of technological knowledge and investments in knowledge production in the form of research and development, and sets before itself the task of estimating the impact of R&D activities on output and growth.

The role of the knowledge production function in this literature is to describe the relationship between knowledge and R&D investment. The KPF is of the

[1] See Griliches (1979) for a discussion of the relative merits of these alternative investigative approaches.

[2] A recent survey of work on the R&D-productivity nexus is Mohnen and Hall (2013). For an overview of studies estimating the rate of return to R&D see Hall, Mairesse and Mohnen (2010). Surveys of literature on spillovers can be found in (Branstetter, 1998) and Cincera and de la Potterie (2001); a more recent survey on this topic is Belderbos and Mohnen (2013).

form: [3]

$$\dot{A} = R \tag{2}$$

which we may term, for referential convenience, the "Griliches knowledge production function." In this equation $\dot{A}$ is the output of new knowledge and R is input into knowledge discovery effort by way of R&D expenditure (Griliches, 1990). [4]

Knowledge stock in the Griliches framework is a cumulation of current and prior additions to knowledge resulting from the stream of R&D expenditures. (In the following discussion we denote stock variables using boldface font.) In the absence of knowledge depreciation, knowledge can be described simply as the sum of the current and past R&D investments:

$$\mathbf{A}_t = \sum_{i=-\infty}^{t} \dot{A}_i = \sum_{i=-\infty}^{t} R_i. \tag{3}$$

However, in deriving knowledge stock from the knowledge production function, knowledge depreciation must be taken into account. Knowledge depreciation, (alternatively termed "R&D depreciation" in the literature), is a phenomenon analogous to depreciation in capital theory (Benhabib and Rustichini, 1991) which features prominently in literature on measuring returns to R&D (Hall, 2007). Because over time knowledge loses its relevance, prior R&D expenditures are thought to contribute less to the current knowledge stock than current expenditures. In the notation provided by Griliches (1979), the stock of technologically relevant knowledge is expressed as a function of the R&D expenditure stream using the following equation:

$$\mathbf{A}_t = G(W(B)R, v). \tag{4}$$

Equation (4) is sometimes called a "knowledge function" (Esposti and Pierani, 2003), although, as we shall see, strictly speaking it is a function describing knowledge accumulation. In this equation, $\mathbf{A}$ is the stock of technological knowledge and $W(B)R$ is an index of current and lagged R&D expenditures. The function $G(W(B)R)$ can be re-expressed as:

$$\mathbf{A}_t = R_t + (1-\gamma)\mathbf{A}_{t-1}, \tag{5}$$

[3]If we include the stochastic component, the KPF is of the form $\dot{A} = R + u$, per Griliches (1990), but inclusion of the stochastic element at the doorstep of our discussion is not essential and would only unnecessarily complicate our analysis.

[4]Griliches (1990) uses K to represent knowledge. The notation has been changed from the original for consistency with the nomenclature used in the rest of this article.

which will be recognized as the perpetual inventory method (PIM) for calculating stock variables. In the PIM equation current stocks are calculated as the sum of current-period investments (R_t) and the stocks left over from the previous period adjusted for depreciation ($(1-\gamma)\mathbf{A}_{t-1}$). The depreciation rate is given by the parameter γ. Further discussion of the relationship between the Griliches KPF and the PIM can be found in Appendix A.

The variable v in equation (4) represents all other factors influencing the stock of knowledge, so that the equation expresses knowledge stock as a function of the sum of the current-period R&D and depreciated R&D from prior periods, plus the residual factors v. These residual factors have not played much of a role in empirical construction of R&D stocks. One review of the literature on R&D an productivity found that "[a]lmost all... have used a simple perpetual inventory or declining balance methodology with a single depreciation rate to construct the knowledge capital produced by R&D investments" (Hall, Mairesse and Mohnen, 2010, p. 15).

It is worthy of notice that while the knowledge production function in equation (2) is at the heart of the Griliches framework, it is not very salient. The aim of this literature is to study the effect of knowledge created by R&D on productivity. Because in the productivity literature equation (2) serves simply as a transition point on the way to calculation of R&D stock, given by the knowledge accumulation equation (4), it is easy to miss.

Knowledge Production and the Theory of Endogenous Growth

The knowledge production function features more prominently in growth theory. Although the knowledge production sector is only one element of a complete endogenous growth model, it is of focal importance, since the growth rate of knowledge determines the growth rates of all other variables in the system. The knowledge production function encountered in Endogenous Growth Theory, which we term the Romer-Jones knowledge production function, is of the form: [5]

$$\dot{A} = \delta L_A^{\lambda} \mathbf{A}^{\phi}, \tag{6}$$

where $\dot{A}$ is knowledge flow, $\mathbf{A}$ is knowledge stock, L_A is labor employed in the R&D sector, λ is a parameter measuring the return of knowledge from R&D labor and ϕ is the intertemporal spillover parameter. This rendition of the knowledge creation process includes knowledge stock ($\mathbf{A}$) on the right-hand side to account

[5] Above is the parametrized KPF adapted from Romer (1990) by Jones (1995). Variations exist, based on slightly different interpretations of the the labor variable, restrictions on parameters λ and ϕ and utilization of a different nomenclature for variables. In Romer (1990), the exact notation used was $\dot{A}=\delta H_A A$; with knowledge represented by A and H_A denoting the amount of human capital. Jones uses the form $\dot{A} = \delta L_A^{\lambda} A^{\phi}$; where knowledge stock is represented by $\mathbf{A}$, L_A is labor employed in R&D, and δ is the arrival rate of innovations. However, these differences are not essential for our analysis. For consistency we have kept to the Jones (1995) notation throughout the whole paper.

for the possibility that knowledge output depends on the stock of already accumulated knowledge.

Even a cursory glance at equations (2) and (6) reveal that the Griliches and Romer-Jones KPF present quite different theories of knowledge formation. In the literature on returns to R&D, knowledge production is synonymous with research effort (Hall, Mairesse and Mohnen, 2010). In endogenous growth theory too, new technologically relevant ideas involve research effort, but the arrival rate of innovations is also conditioned by the stock of previously accumulated knowledge (Romer, 1990; Aghion and Howitt, 1992; Jones, 1995). Furthermore, the measure of research effort in the two models is different. The Romer-Jones knowledge production function proxies research effort using the quantity of labor employed in the R&D sector; Griliches measures research effort with R&D expenditure, which is a broader measure incorporating the labor—and physical capital—components of the knowledge discovery effort. Finally, the two models of knowledge production differ in their approach to accumulation. In the Griliches knowledge accumulation equation research effort accumulates, contributing to the stock of knowledge. In the Romer-Jones model, knowledge accumulates, but research effort does not. The Romer-Jones KPF includes two factors: the existing body of knowledge ($\mathbf{A}$) and the number of scientists and engineers in the R&D sector (L_A). The former is a stock but the latter is a flow variable.

It might appear at first glance that the count of scientists and engineers can be considered a stock variable. This argument does not stand up to close examination. From the standpoint of the production system, labor's contribution to production is through the services it renders to the production process. Whether L_A is a stock or a flow depends on whether it includes labor services rendered in prior periods. If only contemporaneous labor services are included in production, as in the case of Romer-Jones, the labor variable is a flow.[6]

Differences between the Griliches and Romer-Jones conceptions of knowledge production can have profound implications for modeling and empirical estimation. If existing knowledge stock serves as a factor in knowledge production, then its omission from the Griliches KPF will result in omitted variable bias and skew the resulting estimates of R&D elasticity, a point raised previously by Jones and Williams (1998). Omission of accumulated R&D effort from the Romer-Jones KPF can be expected to lead to biases of its own. What, then, is a better way to model knowledge production? What factors should be included in a knowledge production function? In the next section we consider inputs into knowledge production and their inter-relationships. This exercise leads to three observations which serve as a scaffolding on which we build a more general knowledge production function, in Section (IV), of which Griliches and Romer-Jones functions are special cases.

[6]What would make the labor variable a stock is inclusion of prior-period labor services.

III. R&D Capital, Human Capital and Knowledge

Knowledge and R&D are frequently invoked concepts in economics. Yet, in existing literature, the definitions of knowledge and R&D and their inter-relationship are not always made clear, nor are these definitions consistent across studies. Existing literatue has treated research effort and knowledge in one of two ways, either by equating research effort to knowledge or by describing a process by which research effort is turned into knowledge. In the first, "effort as knowledge", perspective the two concepts are either treated as synonymous, or a measure of one is used as a close proxy for the other. A mark of the "effort as knowledge" literature is that the two terms are used interchangeably. Griliches (1992) is one of many examples of this usage. In that study the variable $\mathbf{A}$ in the accumulation equation (4) is referred to as both "knowledge capital" and "R&D capital".

The "effort as knowledge" perspective has been dominant in studies of productivity. This is a perspective hardwired into the Griliches KPF (equation (2)) which sets a sign of equality between $\dot{A}$ and R. In the accompanying PIM accumulation equation (5) R&D turns into knowledge seamlessly. One becomes the other, with adjustment only for depreciation. Aside from the reduction owing to depreciation, the model implies that R&D and knowledge are consubstantiate. In this view, knowledge is nothing more than accumulated R&D expenditure.

In "effort to knowledge" discourse, R&D and knowledge are recognized as distinct concepts. The relationship between these concepts is described as a process by which knowledge arises from research effort. In this framework, the flow of new technological ideas ($\dot{A}$) is driven by the allocation of resources to research. The EGT literature has adopted this perspective, describing a process by which knowledge aises from reseach effort, which is represented by the research labor or human capital component of R&D and measured by the number of scientists and engineers. Because the concepts "knowledge" and "R&D" are so central to the study of innovation, we pause to reflect on them.

In economic theory knowledge has more than one alias. The variable "$\mathbf{A}$" in equations (4) and (6), representing the stock of knowledge, also goes under the names "technology" (Benhabib and Spiegel, 2005; Los and Verspagen, 2000) and "total factor productivity" (Caselli and Coleman, 2006). Changes, or new additions to the body of knowledge, the variable "$\dot{A}$" in (2) and (6), alternate between the labels "technical change" (Griliches, 1988), "technological change" (Verspagen, 1995), "new knowledge" (Abdih and Joutz, 2006), "invention" (Griliches, 1979) and "innovation". Finally, the units of measure into which "the stock of general knowledge" (Branstetter, 1998) can be divided has been discussed in terms of "ideas" (Porter and Stern, 2000), "blueprints" (Grossman and Helpman, 1991), "patents" (Sequeira, 2012), "designs" (Romer, 1990; Branstetter, 1998), "inventions" (Jones, 1995), and, once more, "innovations". The word "innovation" has two senses. It can mean "a novelty", or "an act or process of creating or introducing something new". Both meanings are in currency in the literature, with the former definition used as another term for a unit of measure of knowledge (as, for example, in Porter and Stern (2000), and the later as a description of the process of creation of new knowledge (e.g.: Freire-Seren (2001)). It is important to discern the underlying concept that hides behind the fog of labels. This is the task of the following sub-section.

Observation 1: Knowledge is Distinct from R&D

In discussing the attributes of knowledge the literature stresses two features: nonrivalness and partial excludability. The nonrival nature of knowledge allows multiple agents to use it at the same time. Knowledge is nonrival because it is "disembodied" (Benhabib and Spiegel, 2005), that is, "independent of any physical object" (Romer, 1990). It is partially excludable because even though it can be used by multiple agents, there might be a mechanism through which it might be possible to restrict some agents from using it, as is the case when monopoly on its use is provided through patents. Further, the stock of knowledge has no obvious natural bound; in principle it can grow without limit.

In contrast to the disembodied nature of knowledge, R&D is embodied. The Frascati Manual defines R&D expenditure as consisting of several categories. These include capital expenditures on land and buildings, instruments and equipment and computer software, various labor expenses, and "non-capital purchases of materials, supplies and equipment to support R&D performed" (OECD, 2002, p. 109). Most of these objects are rival and excludable. This is particularly the case of research-related real estate, such as land and buildings that host laboratories, but also of research assets linked to labor. A scientist is a rival asset who cannot be put to work in multiple locations at the same time.

Clearly, ideas and R&D are different. There are two key difference between knowledge and R&D: one has to do with the extent of embodiment and the second, with the emplacement within the innovation process. The first difference is in the extent of embodiment. Knowledge is disembodied; R&D, on the other hand, is embodied. Knowledge resides in replicable patterns—arrangement of human brain neurons, books, media, data and patents; research and development expenditure purchases scientific instruments, raises laboratories and pays the salaries of scientists and engineers. Knowledge is the sum total of useful ideas. R&D is the expenditure made with the aim of discovering new useful ideas, and the assets and activities associated with this expenditure.

Knowledge's situation in the innovation process is also distinct from R&D. R&D is an input into knowledge creation, while knowledge is an output from R&D effort. The relationship between knowledge and R&D can be conceptualized by locating the place of each in the production process. In any given period of time, society has a fixed amount of aggregate output which it can spend on different activities. A fraction of society's output is allocated to R&D. R&D, or "R&D expenditure", are resources devoted to the discovery of new knowledge. The amount of accumulated knowledge is one of the inputs defining the productive capacity of society. Consequently, knowledge produced as a result of research and

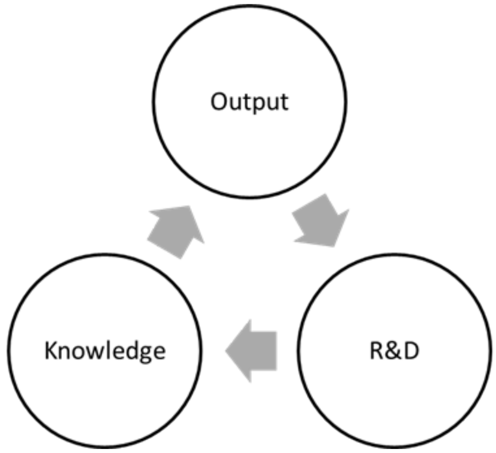

Figure 1. Innovation Cycle

development contributes to economic productivity and increases total output. The "innovation cycle" repeats.

This article insists on conceptual clarity with respect to R&D expenditure and knowledge. Consequently, we maintain a distinction between R&D and knowledge. Maintaining this distinction paves the way for a formal model capturing the essential dynamics of knowledge and R&D.

Observation 2: Knowledge is Distinct from Human Capital

Our next observation pertains the relationship between knowledge and "human capital", the latter understood as the set of human skills and abilities. The distinction between knowledge and human capital is not immediately obvious. For example, is the ability to add, "human capital" or "knowledge"? Foray (2006) looks at knowledge in terms of "expertise", a view which would capture a swath of the territory that rightfully belongs to human capital. By contrast, Romer (1990) has drawn a sharp line between knowledge and human capital. He argues that the former is disembodied and nonrivalrous, while the notion of human capital lacks either attribute. Human capital is not disembodied, he argues, since it is linked to human beings. It is also rivalrous, since a human possessing a skill cannot exercise that skill in multiple places concurrently. Additionally, unlike replication of knowledge, duplication of human skills is not relatively costless: "[t]raining the second person to add is as costly as training the first" (Romer, 1990).

There is no agreement in the literature on where the boundary between knowledge and human capital should lie. While the boundary between knowledge and human capital may be drawn differently, its exact sinuation is not crucial for formal modeling. If a line between two variables can be drawn *somewhere*, we can proceed with a model in which knowledge and human capital are two distinct variables.

Observation 3: "R&D Capital" Includes R&D Labor

Finally, we take care to avoid a misconception that might arise out of the notion of "R&D capital", alternatively referred to in the literature as "knowledge capital", "R&D stock" and "knowledge stock". In the productivity literature it has been widely recognized that R&D expenditures "act as capital", that is, R&D expenditures should be viewed as investments that continue to have an effect post-expenditure and should not be "assumed to be instantaneously depreciated" (Terleckyj, 1980, p. 57). That is why when estimating the elasticity of output with respect to R&D and the rate of return on R&D investment, a measure of "R&D capital" is derived, which consists of cumulated R&D expenditures depreciated at some rate γ.

It is important to observe, however, "R&D expenditures are composed of labor, capital, and material costs" (Hall, Mairesse and Mohnen, 2010, p. 13). Ironically, while in productivity literature the preferred measure of knowledge discovery effort—termed "R&D capital"—also includes labor, it is precisely the capital component of R&D that is excluded from the measure of research effort adopted in Endogenous Growth Theory. Therefore, the term "R&D capital" can be misleading. For this reason we prefer the more neutral term "R&D stock", understood to contain a physical capital component and a labor (or human capital) component.

IV. An R&D-Based Knowledge Production Function

In rethinking the two common forms of the knowledge production function, our research aims to address what we view as the shortcomings of existing frameworks for modeling knowledge creation. The Griliches KPF does not view knowledge as distinct from R&D. It is, however, amenable to modeling an accumulation dynamic. The Romer-Jones functional form has two weaknesses. First, it does not include the full spectrum of effort devoted to knowledge discovery. Embedded in equation (6) is the assumption that discovery effort comes only from labor, excluding physical capital used in R&D.[7] The second shortcoming of the Romer-Jones KPF is that it does not consider accumulation in effort applied to idea creation. Although the Romer-Jones model of the knowledge sector incorporates accumulated knowledge stocks as a factor in knowledge production, it includes only current-period effort devoted to the discovery of new knowledge, which is measured by research labor (L_A). However, from literature on R&D we know that knowledge discovery is subject to lags (Griliches, 1979). Consequently, current-period discovery effort is important, but so is effort made in prior time periods. Because the Romer-Jones KPF sets up research effort as a flow, it is unable to

[7] In Romer (1990) human capital is included as a factor representing research effort; in Jones (1995) knowledge discovery effort comes from labor employed in R&D.

take into account its accumulation. In short, Griliches KPF is a step in a model of knowledge accumulation; the Romer-Jones KPF is mostly about production. But in studying knowledge, innovation, and growth, we are interested in both. A more general model should capture both the production and accumulation of both, knowledge and research effort.

Below we follow up the discussion of KPF functional forms in Section II and the relationship between KPF factor inputs in Section III with an alternative knowledge production function that addresses the weaknesses of earlier approaches. What are the positive recommendations for the construction of this KPF? Observation 1 militates against the tautology between knowledge and R&D of equation (2). From Observation 2 we conclude that an R&D human capital component should be included as a factor in the KPF, distinct from the knowledge stock factor. Observation 3 argues for inclusion of an R&D physical capital variable. Finally, all factor variables should enter the KPF as stocks, in order to account for lagged effects.

In a generic knowledge production function new knowledge is an output related to a list of inputs. We can conceive of a knowledge production process in which new knowledge results from research effort, modulated by the stock of already existing knowledge. Effort devoted to knowledge production is measured by an R&D expenditure variable resulting in a Cobb-Douglas form R&D-based knowledge production function:

$$\dot{A} = \delta \mathbf{R}^{\zeta} \mathbf{A}^{\phi}, \tag{7}$$

where δ is the productivity parameter, $\mathbf{A}$ is knowledge stock, and $\mathbf{R}$ represents accumulated R&D stock.[8] Parameter ζ measures the elasticity of knowledge with respect to R&D. The intertemporal knowledge spillover parameter ϕ measures the contribution of extant knowledge stock to the production of new knowledge.

$\mathbf{R}$ is a composite input consisting of human and physical capital stocks:

$$\mathbf{R} = \mathbf{L}_A^{\lambda} \mathbf{K}_A^{\kappa} \tag{8}$$

where $\mathbf{L}_A$ denotes the accumulated effort of labor employed in the R&D sector, $\mathbf{K}_A$ is physical capital devoted to R&D, and parameters λ and κ represent the share of each factor in total R&D. This formulation diverges from the EGT assumption that human capital inputs of labor are sufficient proxy for research effort. It might be argued that in the R&D sector labor inputs are most of what matters for knowledge creation because of the assumed high share of labor in R&D expenditure. Nevetheless, creation of new knowledge requires research labs

[8] A similar KPF formulation is found in Jones and Williams (1998) with the difference that in that study the authors measure research effort with the flow of R&D. The Jones and Williams (1998) KPF can be expressed as $\dot{A} = \delta R^{\zeta} \mathbf{A}^{\phi}$ using the boldface notation to differentiate between flow and stock variables.

as well as research lads. In fact, R&D is more capital-intensive than the productive sector. Cross-country data on R&D presented in Table (A1) show that the non-labor share in R&D is not negligible and therefore should not be discarded uncritically. Recognition that research effort is a composite input consisting of both human and physical capital components should be expected to have the practical effect of correcting an omitted variable bias in empirical estimates.

Substituting (8) into (7) results in an *extended R&D-based knowledge production function*:

$$\dot{A} = \delta\Big(\mathbf{L}_A^{\lambda}\mathbf{K}_A^{\kappa}\Big)^{\zeta}\mathbf{A}^{\phi}. \tag{9}$$

We can expect the accumulation of labor and capital employed in R&D to proceed differently from each other. For example, the rates of growth and depreciation of labor and capital aggregates will almost surely not be identical. Such differences can be accounted for, and they will be considered in our framework in Sections IX and X.

The R&D-based knowledge production function bridges the gap between alternative formulations of knowledge dynamics coming from EGT and the R&D productivity literatures. We note that the Romer-Jones KPF is a special case of the proposed KPF, under the restrictions $\lambda = 1$ and $\kappa = 0$ and with lags of $\mathbf{L}_A$ discounted at a 100% rate. Likewise, the Griliches KPF is a special case of equation (9) under the restrictions $\delta = 1$, $\phi = 0$, $\zeta = 1$ and 100% depreciation for lags of $\mathbf{K}_A$ and $\mathbf{L}_A$.

The R&D-based knowledge production function forms the core of our model of knowledge dynamics. But, by itself, the knowledge creation process is insufficient to explain the full range of knowledge dynamics since it does not take into account the accumulation of the stocks that serve as factors of production. What remains is to embed the knowledge production equation within a framework of accumulation. That is the task of the next two sections.

V. Building Blocks of a Knowledge Dynamics Model

In developing a model of the knowledge sector we embed the R&D-based knowledge production function in a broader framework. This section presents, in general terms, the elements of this framework. Our knowledge dynamics model consists of four components. The first, is a rule by which investments are allocated to the R&D sector. By this rule a stream of flows into the R&D sector is generated. The second module is a process of R&D accumulation, that takes into account depreciation, or obsolescence, of aged R&D stocks. A knowledge production process represented by a KPF is the third component. The fourth module is a model of knowledge accumulation, which works similarly to the R&D accumulation process. In this and following sections, wherever expositional simplicity is desired, we work with the more compact form of the R&D-based KPF (equation (7)), relying on the **R** composite.

R&D investment

Knowledge creation begins with provision of resources for research. In each time period some economic resources are allocated towards research and development. This incremental addition to R&D ("R&D increment") is described by an ***R&D investment equation*** the general form of which is:

$$R_{I_t} = F_{R_I}(E(t, ...)), \tag{10}$$

where R_{I_t} represents the economic resources devoted to R&D, t is the time index, and E is a vector of variables that determine R_I. The allocation of resources for R&D can be made on the basis of a fixed proportion of total resources (i.e. a set percentage of GDP) or follow from some other allocation rule. One can imagine a number of societal R&D investment rules. In principle, we can treat R_I as constant, as a variable growing at a constant rate, or as a variable governed by a more complex functional form. For example, R_{I_t} can be derived from a profit-maximization rule. Current-period R&D investment could also be formulated to depend on prior-period R&D ($R_{I_{t-1}}$), or on past or current macroeconomic conditions (Y).

R&D stock accumulation

Next, we turn to consider the accumulation dynamics of R&D stock. R&D accumulation consists of two processes: investment and depreciation. The stock of R&D increases as a result of of R&D investment. At the same time, the R&D stock is subject to depreciation. The law of motion for R&D is described by the ***R&D stock accumulation equation***:

$$\dot{R} = R_I - \gamma_R * \mathbf{R}, \tag{11}$$

where $\dot{R}$ represents the net change in R&D stock, R_I is the incremental addition to R&D, R is extant R&D stock, and γ_R is the R&D depreciation rate.

Knowledge production

Current-period incremental increase in knowledge is described by a ***knowledge production function*** F_A, the general form of which is:

$$A_I = F_A(O(...)), \tag{12}$$

where $O(...)$ is a list of factors of knowledge production. Two competing forms of this function were considered in Sections (II) and (IV), and a general form of a knowledge production equation was proposed in Section (IV). Below we will work with the more general R&D-based KPF, $A_I = \delta \mathbf{R}^{\zeta} \mathbf{A}^{\phi}$. The R&D stock variable is present in the KPF, as well as in the R&D accumulation equation, a feature that allows closure of the full model of the knowledge sector.

The amount of knowledge created in a society at any given time can be conditioned by factors not explicitly considered here. For example, the suitability of the general political and economic environment for innovation, quality of institutions, and the intellectual property regime, can be expected to influence the arrival rate of innovations. Such variables could also be included as factors in an R&D-based KPF. Because these aspects of knowledge production are not the focus of this inquiry, they are not explicitly included in the full model. The residual term δ of the R&D-based knowledge production function is a catch-all for other determinants of the innovation arrival rate.

Knowledge stock accumulation

Much like R&D stock and physical capital, knowledge too has been theorized to exhibit accumulation dynamics, that is, being subject to creation and depreciation (Griliches, 1990). The notion of depreciation in the context of physical capital is based on the physical phenomena of wear and breakdown. For R&D stock, the meaning of depreciation is linked to the wear and tear of equipment used in research (depreciation of physical capital employed in research), as well as obsolescence of capabilities embodied in humans working on the creation of new ideas. In the context of knowledge accumulation, the concept of depreciation relates to obsolescence of ideas in their capacity to contribute to the creation of new ideas.[9]

Mathematically, the treatment of accumulation dynamics in knowledge stock is identical to that of R&D stock. Accumulated knowledge stock can be defined as the sum of all additions to knowledge, adjusted for depreciation. In each time period, the change in knowledge stock $\dot{A}$ is determined by the amount of knowledge currently produced (A_I) minus depreciated stock. The evolution of knowledge stock is described by a ***knowledge stock accumulation equation***:

$$\dot{A} = A_I - \gamma_A * \mathbf{A} \tag{13}$$

Dividing the above knowledge stock accumulation equation by A we get the proportional growth rate for knowledge, $\frac{\dot{A}}{\mathbf{A}} = \frac{A_I}{\mathbf{A}} - \gamma_A$, that is, the growth rate of the knowledge stock is the difference in the rate of creation of new knowledge ($\frac{A_I}{A}$) and the rate of obsolescence of the extant knowledge stock (γ_A). Combining knowledge production with knowledge accumulation we get the following general form for the law of motion of knowledge stock:

[9]Note that an obsolete idea can continue to be useful in the physical economy while losing its capacity to contribute to creation of new ideas. A bicycle, once invented, can continue to be manufactured and used while yielding its significance to more cutting edge technological innovations in transportation.

$$\dot{A} = F_A(O(...)) - \gamma_A * \mathbf{A} \tag{14}$$

The knowledge stock in any period is the result of accretion of the above knowledge flows. Thus, integrating $\int_{-\infty}^{m} \dot{A}(t)\, dt$ will give us knowledge stock at time m.

VI. A Model of Knowledge Dynamics

We build the model by giving concrete functional forms to the four building block equations previously specified in implicit form. The building block equations can then be integrated into a complete model of knowledge and R&D dynamics. A full model will show the state of knowledge and R&D stocks at any given point in time. It will also reveal the short-term and long-term growth rates of the two stocks and their sensitivity to parameters in the production and accumulation equations. With proportional growth rates in hand it will be possible to address the question whether a double-stock model with accumulation of knowledge and R&D is consistent with balanced growth.

In the remainder of the article, whenever all variables are explicitly defined, so that it is easy to distinguish variables representing stocks from those indicating flows, we dispense with the convention of using boldface font to designate stock variables.

R&D stock and growth

Let us assume that in every time period, a certain amount of resources, $R_I(t)$, is allocated towards research and development. Let us further assume that this R&D increment starts from a base value of R_{I0} in time period $t = 0$ and grows over time at a rate θ_R:

$$R_I(t) = R_{\mathrm{I0}} e^{t\theta_R}. \tag{15}$$

The above R&D investment equation defines the stream of R&D flows. Initially, we place no restrictions on the growth rate θ_R.[10]

R&D stock evolves according to the previously discussed R&D stock accumulation equation:

$$\dot{R}(t) = R_I(t) - \gamma_R * R(t).$$

[10] The growth rate θ_R can be any real number and need not be constant. If the growth rate is time-dependent ($\theta_R[t]$) then we are avoiding an imposition of a specific functional form (exponential) for the growth rate of R_I; this amounts to stating that at any given time we can extrapolate an exponential growth rate, but the actual change in R_I can be governed by a generating process described by another functional form.

We can obtain the formula for R&D stock in two steps. The first step is to substitute the R&D investment equation into the R&D stock accumulation equation:

$$\dot{R} = R_{\mathrm{I}0} e^{\theta_R * t} - \gamma_R * R, \tag{16}$$

which yields the law of motion for R as an explicit differential equation. In step two, solving equation (16) for R produces an equation for R&D stock:

$$R(t) = \frac{R_{\mathrm{I}0} e^{\theta_R * t}}{\gamma_R + \theta_R} + e^{-t\gamma_R} C_1 \ , \tag{17}$$

where C_1 is a constant of integration. In the long run, as $t \to \infty$, the term with the contast of integration approaches zero.

As we would expect, the greater the depreciation rate γ_R, the lower the R&D stock. R&D stock is also positively dependent on the size of the initial R&D increment (R_{I0}) and the growth rate of the R&D increment (θ_R). In the case of θ_R, the exponent θ_R predominates over the term in the denominator, so on the whole the long-run effect of a high growth rate of the increment is positive. We can see from the derivative of R with respect to the growth exponent θ_R (equation (18)) that the positive term predominates for arbitrarily large values of t:

$$\partial_{\theta_R} R(t) = R_{\mathrm{I}0} \left(e^{t\theta_R} \right) \left(\frac{t}{\gamma_R + \theta_R} - \frac{1}{(\gamma_R + \theta_R)^{\,2}} \right). \tag{18}$$

In the special case where $\theta_R = 0$ (and setting the term with the constant of integration to zero) long-run capital stock simplifies to $R(t) = \frac{R_{\mathrm{I}0}}{\gamma_R}$.[11]

Taking the derivative of $R(t)$ in equation (17) gives us the equation for growth of R&D stock:

$$\dot{R}(t) = \theta_R \frac{R_{\mathrm{I}0} e^{t\theta_R}}{\gamma_R + \theta_R} - \gamma_R e^{-t\gamma_R} \ C_1. \tag{19}$$

The proportional growth rate for R is:

$$\frac{\dot{R}(t)}{R(t)} = \frac{\theta_R \frac{R_{\mathrm{I}0} e^{t\theta_R}}{\gamma_R + \theta_R} - \gamma_R e^{-t\gamma_R} \ C_1}{\frac{R_{\mathrm{I}0} e^{t\theta_R}}{\gamma_R + \theta_R} + e^{-t\gamma_R} C_1}. \tag{20}$$

[11] Note that $\theta_R = 0$ does not imply that the R&D increment is zero. Under this assumption only the growth rate of the increment is zero, while the R&D investments are a constant stream equaling $R_{\mathrm{I}0}$ in each period.

If we let the terms with constants of integration equal zero the proportional growth rate for R&D stock R reduces to the growth rate of the R&D increment θ_R:

$$\frac{\dot{R}(t)}{R(t)} = \theta_R. \tag{21}$$

Although growth of R&D stock $\dot{R}(t)$ depends on θ_R, γ_R, and R_{I0}, the proportional growth rate is a function only of θ_R. Particularly interesting is that the depreciation rate γ_R drops out of the proportional growth rate. That is because both the numerator $\dot{R}(t)$ and the denominator $R(t)$ are subject to depreciation, consequently the γ_R cancels out.

Let us stop to consider growth in R&D stock along a balanced growth path. A balanced growth path is an idealized scenario when all macroeconomic variables grow at a constant rate. Let us assume that the economy is growing at such a constant rate θ_Y^*. If the economy allocates a fixed percentage of its total output to R&D, the growth rate of the R&D increment will equal the growth rate of output:

$$\theta_R^* = \theta_Y^*. \tag{22}$$

In equation (21) above we have shown that the rates of growth of R&D increment equals growth of the overall R&D stock. It follows that along the balanced growth path R&D stock will increase at the same rate as aggregate output:

$$\frac{\dot{R}(t)}{R(t)} = \theta_Y^*. \tag{23}$$

Under the assumption that the proportion of output allocated to R&D remains fixed the model is consistent with balanced growth. The fixed-proportion assumption can be justified on theoretical grounds as arising from the logic of balanced growth. It is also in line with business and policy practice, supported by the observation that firms follow a fixed-proportion heuristic in budgeting for R&D, which is adjusted infrequently. Furthermore, at the level of national policy governments often commit to spend a target proportion of GDP on research.

Knowledge stock and growth

Armed with the information on R&D stock accumulation, we now consider the production and accumulation of knowledge stock, in which the former plays a key part. The incremental addition to knowledge stock (A_I) is determined by the R&D-based knowledge production function:

$$A_I(t) = \delta * (R(t))^{\zeta} * (A(t))^{\phi}. \tag{24}$$

Substituting the KPF into the knowledge stock accumulation equation we arrive at the following law of motion for knowledge stock:

$$\dot{A}(t) = \delta * (R(t))^{\zeta} * (A(t))^{\phi} - \gamma_A * A(t). \tag{25}$$

Because our focus is on the long-term dynamics, in further analysis we omit the term related to the constant of integration, which, as mentioned previously, equals 0 in the limit, as $t \to \infty$. The solution of the differential equation (25) gives us knowledge stock:

$$A(t) = \left(\frac{\delta \left(\frac{R_{I0} e^{t\theta_R}}{\gamma_R + \theta_R} \right)^{\zeta} (1-\phi)}{\zeta\theta_R + \gamma_A(1-\phi)} \right)^{\frac{1}{1-\phi}} \tag{26}$$

The rate of change in the stock of knowledge is:

$$\dot{A}(t) = \frac{\zeta\theta_R \left(\frac{\delta \left(\frac{R_{\mathrm{I}0} e^{t\theta_R}}{\gamma_R + \theta_R} \right)^{\zeta} (1-\phi)}{\zeta\theta_R + \gamma_A(1-\phi)} \right)^{\frac{1}{1-\phi}}}{1-\phi} \tag{27}$$

and the proportional growth rate is:

$$\frac{\dot{A}(t)}{A(t)} = \frac{\zeta\theta_R}{1-\phi}; \phi \neq 1. \tag{28}$$

Knowledge grows at a rate proportional to the growth rate of the R&D investment increment θ_R. There is a positive relationship between proportional growth rate of knowledge, the R&D growth parameter θ_R, the R&D stock elasticity of knowledge parameter ζ, as well as the intertemporal elasticity of knowledge ϕ. Because equation (28) is undefined at the point $\phi = 1$, we need to impose a technical restriction $\phi \neq 1$.

If the model of the real economy is defined as in Romer (1990), along a balanced growth path the growth rate of knowledge will pin down the growth rate in other variables, including output, so that:

$$\frac{\dot{Y}(t)}{Y(t)} = \frac{\dot{R}(t)}{R(t)} = \frac{\dot{A}(t)}{A(t)}. \tag{29}$$

Along a balanced growth path, as output, R&D increment and R&D stock grow at the same rate θ_R^*, the proportional growth of the knowledge stock will equal:

$$\frac{\dot{A}(t)}{A(t)} = \frac{\zeta\theta_R^*}{1-\phi} = \theta_Y^*; \phi \neq 1. \tag{30}$$

Equations (29) and (30) are reconciled under the following additional restriction:

$$\zeta + \phi = 1. \tag{31}$$

To conclude, in a double-stock model, balanced growth equilibrium implies constant returns to scale for the two inputs of knowledge production.

VII. Comparative Statics of Knowledge Stock

In an endogenous growth model the technological capability of the economy is set by the accumulated stock of knowledge. Knowledge stock, in turn, is determined jointly by time and the knowledge and R&D production and accumulation parameters: ζ, ϕ, θ_R, γ_A, γ_R, and R_{I0}. It is therefore of interest to determine the sensitivity of the technology level to the parameters. This aspect of technology can be captured by the elasticity of knowledge stock (A) to the various parameters of the knowledge accumulation model. The elasticity of Y with respect to X represents the percentage change in variable Y as a result of a percentage change in variable X, for which we adopt the notation σ_{YX}.

In the model presented, R&D stock is the only real-economy factor involved in the creation of knowledge stock. The elasticity of A with respect to R represents the percentage increase in the technological sophistication of the economy in response to a percentage increse in R&D stock, and can be shown to be:

$$\sigma_{AR} = \frac{\zeta}{1-\phi}. \tag{32}$$

The sensitivity of knowledge stock to R&D stock is independent of various features of accumulation, such as rates of depreciation γ_R and γ_A and the increment growth rate θ_R. The elasticity σ_{AR} depends only on two parameters of knowledge production: ζ and ϕ. The elasticity σ_{AR} is positively related to ζ and also to ϕ, provided that $\phi < 1$.

The relationship between knowledge stock and the rate of investment in R&D

is captured by the elasticity of A with respect to θ_R:

$$\sigma_{A\theta_R} = \frac{\zeta\theta_R(t-1) - (\frac{1}{(\gamma_R+\theta_R)})}{(1-\phi)}. \tag{33}$$

The elasticity parameter $\sigma_{A\theta_R}$ depends positively on ζ, $\phi < 1$, and the R&D depreciation rate γ_R. The sensitivity of A to changes in θ_R varies with θ_R itself; the greater θ_R, the higher is the elasticity. Furthermore, the elasticity is time-dependent, increasing with the progression of time. The latter finding can be understood as a result of the cumulative effect of a change in the R&D growth rate.

As can be expected, knowledge stock is negatively affected by depreciation. The change in A in response to change in γ_R is given by the equation for $\sigma_{A\gamma_R}$:

$$\sigma_{A\gamma_R} = -\frac{\zeta\gamma_R}{(\gamma_R+\theta_R)(1-\phi)}. \tag{34}$$

Higher values for knowledge production parameters ζ and $\phi < 1$ increase the absolute value of the elasticity of knowledge with respect to the R&D depreciation rate. An increase in the R&D growth rate θ_R, on the other hand, reduces the absolute value of $\sigma_{A\gamma_R}$. Elasticity $\sigma_{A\gamma_R}$ tends to be negative under realistic assumptions for values of the other parameters in equation (34). For example, if γ_R, θ_R, and ζ are greater than zero and $0 \le \phi < 1$, $\sigma_{A\gamma_R}$ is negative—meaning that an increase in the depreciation rate of R&D leads to a lower knowledge stock. A similarly negative relationship obtains between knowledge stock and the knowledge depreciation rate γ_A:

$$\sigma_{A\gamma_A} = -\frac{\gamma_A}{\zeta\theta_R + \gamma_A(1-\phi)}. \tag{35}$$

Under the above assumptions regarding the values of θ_R, ζ, ϕ, and assuming, furthermore, that γ_A is positive, $\sigma_{A\gamma_A}$ will be less than zero. The elasticity of knowledge stock with respect to the knowledge stock depreciation rate abates, in absolute value terms, at higher values of ζ and θ_R. More robust pace of allocation of new resources for research (reflected in higher θ_R) and greater productivity of R&D resources in the generation of new knowledge (observed as higher ζ) ameliorate the negative effects of knowledge depreciation on the technology level. When $\phi < 1$, higher values of ϕ have an opposite effect, leading to greater elasticity of knowledge stock to knowledge depreciation.

VIII. R&D Accumulation Dynamics and the Measurement of the Elasticity of Innovation

Having considered the influence of the several model parameters on the technology level in Section (VII), we move to investigate the implications of our model of knowledge dynamics for the modeling and measurement of innovation. While technology level is represented in our model by knowledge stock A, innovation, understood as the rate of arrival of new ideas, or alternatively, "knowledge flow", is represented by $\dot{A}$.[12] The chief question is whether a model of knowledge dynamics that takes into account the accumulation of knowledge and R&D paints the same picture of the way factors to knowledge production influence innovation.

The two factors of knowledge production are R&D (R) and knowldge (A) and the impact of a factor on innovation output is captrued by the notion of output elasticity. The elasticity of innovation with respect to R&D ($\sigma_{\dot{A}R}$) and knowledge ($\sigma_{\dot{A}A}$) provide information on the percentage increase in innovation output in response to a percentage increase to the the respective factor. The R&D elasticity of innovation, in particular, has been the subject of some attention in the literature because R&D is seen as the primary way of enhancing innovative performance and growth.

In the context of our model of knowledge dynamics we can ask what would be the R&D elasticity of innovation ($\sigma_{\dot{A}R}$) if we measure R&D as a stock and whether that estimate will be different from an elasticity based on a measure of R&D as a flow, as has been the method in the literature. We can recall that in our model R&D stock is defined as per equation (17). As $t \to \infty$, the expression for long-run R&D stock simplifies to:

$$R(t) = \frac{R_{\mathrm{I}0} e^{\theta_R * t}}{\gamma_R + \theta_R}. \tag{36}$$

We denote research flow with $\dot{R}$ to differentiate it from research stock. If we measure research effort with the stream of current R&D flows, research effort will be given by the equation:

$$\dot{R}(t) = R_{\mathrm{I}0} e^{\theta_R * t}. \tag{37}$$

Now, what would be the implication of measuring R&D elasticity of innovation with flows as opposed to stocks? It turns out that the consequences can be serious.

The elasticity of innovation with respect to R can be derived from equations (27) and (36):

[12] Recall the theoretical discussion in Section (III).

$$\sigma_{\dot{A}R} = \frac{\zeta^2 \theta_R}{(1-\phi)^2}. \tag{38}$$

The elasticity of innovation is positively related to parameters θ_R, ζ and ϕ. This is the "true" R&D elasticity of innovation for a knowledge generation dynamic process in which earlier research expenditures contribute to current-period knowledge production.

Elasticity of innovation with respect to $\dot{R}$ is obtained by replacing R with $\dot{R}$ in the differential equation (25) and solving for A. Subsequently, $\dot{A}$ can be derived, which in combination with equation (37) yields the expression:

$$\sigma_{\dot{A}\dot{R}} = \zeta^2 \theta_R. \tag{39}$$

The above formula for R&D elasticity of innovation obtains under the assumption that only current-period R&D expenditures figure as factors in the production of new ideas. If this assumption does not hold and research expenditures contribute to innovation output with a lag, $\sigma_{\dot{A}\dot{R}}$ will be a biased estimate of the actual impact of R&D expenditure on innovation. The magnitude of this distortion will be given by the difference between the true and biased elasticity:

$$\sigma_{\dot{A}R} - \sigma_{\dot{A}\dot{R}} = \theta_R \frac{\zeta^2 (2-\phi)\phi}{(\phi-1)^2}. \tag{40}$$

The bias will be zero if the R&D increment is constant ($\theta_R = 0$), if R&D spending has no role in knowledge creation ($\zeta = 0$) or if intertemporal spillovers are absent ($\phi = 0$). However, under more realistic scenarios there will be a bias such that $\sigma_{\dot{A}\dot{R}}$ will tend to underestimate the true elasticity. The structure of the bias is determined by the technology parameters ζ and ϕ, but its magnitude is scaled by the R&D growth rate θ_R. In many scenarios the degree of underestimation will be extreme.

Figure 2 plots the measurement bias for various combinations of parameters ζ, ϕ and R&D growth rate θ_R. The four contour plots in the figure represent different assumptions about the growth rate θ_R. The shading of the areas between the contours represents the magnitude of the distortion, with lighter shading indicating greater magnitudes. The black triange in each plot represents the $\zeta - \phi$ parameter space for which the magnitude of distortion is modest, 5% or less. The white triangles show combinations of parameters for which $\sigma_{\dot{A}\dot{R}}$ underestimates $\sigma_{\dot{A}R}$ by 200% or more. In general, the degree of distortion is higher for higher values of ζ and ϕ. Bias is also greater when R&D investment grows rapidly. When $\theta_R = 5\%$ the magnitude of distortion is 5% or less for slightly more than half of the $\zeta - \phi$ parameter space. This is a slight consolation since significant distortions will still predominate in a large slice of the parameter space. Consid-

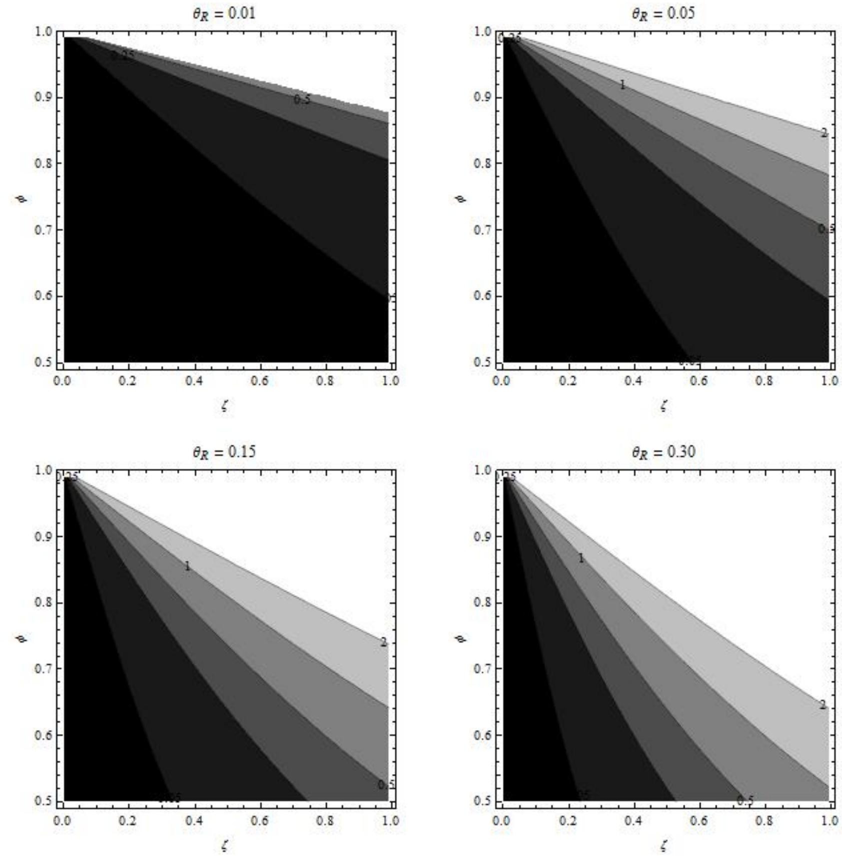

Figure 2. Magnitude of Distortion in the Measure of $\sigma_{\dot{A}R}$

Note: Magnitude of distortion in the measuer of R&D elasticity of innovation is expressed in percentage points, 1=100%.
Source: Authors' calculation.

erable underestimation can be expected even for moderate R&D growth of 5%; in this scenario large bias will result for the vast majority of parameter combinations. When R&D grows at a rapid pace, measurements of the R&D elasticity of innovation with R&D flows as opposted to R&D stocks will result in huge underestimation of elasticity.

IX. R&D Accumulation Dynamics and Econometric Estimation

The use of short-term R&D flows in place of R&D stocks is also problematic from the perspective of econometric estimation. Such substitution can be expected to lead to mis-estimation of the ζ parameter—which the previous section has shown to be one of the determinants of overall R&D elasticity of innovation—further biasing the elasticity estimate. Suppose we try to estimate the knowledge accumulation equation (25), which we reproduce below in a slightly modified form, having incorporated the depreciation term into the dependent variable:

$$\left(\dot{A}(t)+\gamma_A * A(t)\right)=\delta *(R(t))^{\zeta} *(A(t))^{\phi}. \tag{41}$$

On the surface this looks similar to the Romer-Jones KPF in equation (6). The difference between the two equations is in the R variable and its content. In our R&D-based KPF R consists of labor and physical capital components of R&D, measured as stocks:

$$\begin{aligned}
R(t) &= \mathbf{K}(t)^{\kappa}\mathbf{L}(t)^{\lambda} \\
&= \left(\frac{K_{\mathrm{I}0}e^{\theta_K * t}}{\gamma_K+\theta_K}\right)^{\kappa}\left(\frac{L_{\mathrm{I}0}e^{\theta_L * t}}{\gamma_L+\theta_L}\right)^{\lambda} \\
&= \left(\frac{\dot{K}(t)}{\gamma_K+\theta_K}\right)^{\kappa}\left(\frac{\dot{L}(t)}{\gamma_L+\theta_L}\right)^{\lambda}
\end{aligned} \tag{42}$$

In the Romer-Jones KPF only current-period labor flows $\dot{L}$ are included as input in knowledge production besides knowledge stock. We can separate $R(t)$ into an included and excluded component:

$$\begin{aligned}
R(t) &= \dot{L}(t)\mathbf{X}(t) \\
&= \dot{L}(t)\left(\frac{\dot{K}(t)}{\gamma_K+\theta_K}\right)^{\kappa}\left(\frac{1}{\gamma_L+\theta_L}\right)^{\lambda}\dot{L}^{\lambda-1}.
\end{aligned} \tag{43}$$

The variable $\mathbf{X}$ consists of adjustment of the labor input for accumulation, as well as the entirety of physical capital stock involved in R&D.

Econometric estimation of (41) will typically involve linearization through logarithmic transformation. The log-transformed equation can then be estimated via least squares as:

$$\begin{aligned}
\ln\left(\dot{A}(t)+\gamma_A * A(t)\right) &= \ln(\delta)+\phi\ln(A(t))+\zeta\ln\left(\bar{L}(t)\mathbf{X}(t)\right)+\epsilon \\
&= C+\phi\ln(A(t))+\zeta\ln\left(\bar{L}(t)\right)+\zeta\ln\left(\mathbf{X}(t)\right)+\epsilon,
\end{aligned} \tag{44}$$

where C is constant and ϵ is the stochastic error term.

Regression analysis on the basis of current R&D labor flows is tantamount to exclusion of the composite $\mathbf{X}$ variable. As with any omitted variable, the expectation of the ζ estimate might be biased. In this case the form of the estimator of ζ will be given by:

$$\zeta^* = \hat{\zeta} + \hat{\zeta}\frac{\hat{\text{Cov}}(\ln(\bar{L}), \ln(\mathbf{X}))}{\hat{\text{Var}}(\ln(\bar{L}))}, \tag{45}$$

and its expectation as:

$$E[\zeta^*] = \zeta + \zeta\frac{\text{Cov}(\ln(\bar{L}), \ln(\mathbf{X}))}{\text{Var}(\ln(\bar{L}))}, \tag{46}$$

which can in turn be expressed as:

$$E[\zeta^*] = \zeta(1 + \rho), \tag{47}$$

where ρ is the ratio of the covariance between the log of labor flows and the log of the excluded component $\mathbf{X}$ over the variance of the log of labor flows. The sign and magnitude of ρ will determine the direction and degree of distortion of the expected value of the estimator of ζ. If $\rho = 0$, the estimate will not be biased; with $\rho > 0$, then ζ will be overestimated; and if $\rho < 0$ then $E[\zeta^*]$ will be an underestimate of true ζ. It can be shown that ρ is determined by the relationship between labor flows and the omitted variables, so that:

$$\begin{aligned} \rho = {} & \kappa\frac{\text{Cov}\left(\ln\left(\bar{L}\right), \ln\left(\bar{K}\right)\right)}{\text{Var}\left(\ln\left(\bar{L}\right)\right)} - \kappa \\ & - \lambda\frac{\text{Cov}\left(\ln\left(\bar{L}\right), \ln\left(\gamma_L + \theta_L\right)\right)}{\text{Var}\left(\ln\left(\bar{L}\right)\right)} - \kappa\frac{\text{Cov}\left(\ln\left(\bar{L}\right), \ln\left(\gamma_K + \theta_K\right)\right)}{\text{Var}\left(\ln\left(\bar{L}\right)\right)}. \end{aligned} \tag{48}$$

If the growth and depreciation rates $\theta_K, \theta_L, \gamma_K, \gamma_L$ are constant, ρ simplifies to:

$$\rho = \kappa\left(\frac{\text{Cov}\left(\ln\left(\dot{L}\right), \ln\left(\dot{K}\right)\right)}{\text{Var}\left(\ln\left(\dot{L}\right)\right)} - 1\right) \tag{49}$$

Because variance is always positive, the sign of the covariance between labor flows and capital flows will determine the direction of estimation bias. If the covariance between labor flows and capital flows is negative, as would be the case if one input is used to substitute for the other, ζ^* is guaranteed to be an underestimate. If covariance has positive sign the magnitude of bias will depend on the ratio of the covariance term to the variance of labor. If $\text{Cov}\left(\ln\left(\dot{L}\right), \ln\left(\dot{K}\right)\right) > \text{Var}\left(\ln\left(\dot{L}\right)\right)$, as might be expected if R&D labor and physical capital are complements, then $\rho > 0$, leading to empirical overestimation of ζ^*. In the unlikely case that the covariance term is positive and equal in magnitude to the variance of labor, the estimate of the ζ^* parameter will be accurate. All estimation biases can be avoided by correctly specifying the knowledge production function.

X. Omission of Physical Capital and its Implication for Balanced Growth

The above model of the knowledge sector embeds the R&D-based knowledge production function in a framework of double-stock accumulation. Having shown the significance of the model to accurate measurement and estimation of knowledge dynamics, we note a special implication of the model for balanced growth.

In his seminal endogenous growth model, Romer (1990) assumes a knowledge production function of the form:

$$\dot{A} = \delta H_A A \tag{50}$$

where $\dot{A}$ is growth in knowledge, H_A is human capital (assumed to be working in the research sector), A is knowledge stock, and δ is the arrival rate of innovations. This specification for knowledge production implies a proportional growth rate of knowledge:

$$\frac{\dot{A}}{A} = \delta H_A. \tag{51}$$

Growth of knowledge—and of all other variables—is determined by the level of human capital H_A. One implication of the Romer knowledge production function is that ideas-driven growth is possible even in the absence of growth in human capital or growth in population working in research.

Critical of the "scale effects" implications of the Romer model, Jones (1995) proposes a parametrized knowledge production function

$$\dot{A} = \delta L_A^\lambda A^\phi \tag{52}$$

where L_A is labor employed in the research sector, λ is a parameter measuring the return of knowledge from R&D labor and ϕ is the intertemporal spillover parameter. The functional form adopted by Romer is similar to the Jones knowledge production function, but with parameters λ and ϕ set to 1. Jones' knowledge production function implies that along the balanced growth path, the proportional growth rate of kowledge will be:

$$\frac{\dot{A}}{A} = \frac{\lambda\left(\frac{\dot{L_A}}{L_A}\right)}{1-\phi}. \tag{53}$$

The Jones function eliminates scale effects but it also means that to have ideas-driven growth it is not enough to have non-zero employment in the research sector. Contrary to Romer, in Jones (1995), long-term growth in knowledge requires employment in the R&D sector to grow as well.

These examples from the literature illustrate that the specification of the knowledge production function is not trivial. Different functional forms can lead us to widely different conclusions. Variations on the Romer-Jones knowledge production function has been widely used in endogenous growth literature (Freire-Seren, 2001; Sequeira, 2012) and in other research on innovation (Porter and Stern, 2000).

Our model is based on a knowledge production function:

$$\dot{A} = \delta R^{\zeta} A^{\phi} \tag{54}$$

where R is R&D stock and ζ is the R&D to knowledge parameter. R&D stock, R, is the sum of lagged and depreciated R&D effort subsuming human- and physical-capital components. Replacing the R composite with its physical and human capital components gives equation (9), which we reproduce here for convenience:

$$\dot{A} = \delta (L_A^{\lambda\zeta} K_A^{\kappa\zeta}) A^{\phi}. \tag{55}$$

From the point of view of long-term growth, and for comparison with equations (51) and (53), one might be curious about the implications of the above form for idea-based growth. Dividing both sides of (55) by A and taking the time-derivative gives us the following balanced path growth rate for knowledge stock:

$$\frac{\dot{A}}{A} = \frac{\zeta\left(\lambda\frac{\dot{L}_A}{L_A} + \kappa\frac{\dot{K}_A}{K_A}\right)}{1-\phi}. \tag{56}$$

In the long run, the proportional growth rate of knowledge and the economy as a whole is a weighted average of the proportional growth rates of labor and physical capital in the R&D sector multiplied by the term $\frac{\zeta}{1-\phi}$. This model is not characterized by scale effects; at the same time zero growth in L_A does not automatically imply dissipation in the growth rate of ideas. Strictly speaking, idea-based growth is possible even when labor growth is zero. In the case when $\frac{\dot{L}_A}{L_A} = 0$ idea-based growth can be fueled exclusively by the accumulation of research physical capital. Since the R&D sector is fairly capital-intensive[13] an R&D-based model would be incomplete without R&D physical capital.

We argue that equations (54) and (55) provide a more complete model of the knowledge sector. The difference from Romer-Jones is not only in the inclusion of an important omitted variable, K_A, but also in the incorporation of R&D accumulation dynamics. Variables R and A represent stocks that can grow and

[13] Table A1 shows that in most countries physical capital constitutes a substantial share of R&D expenditures. The capital-intensity of the R&D sector has also been noted by Porter and Stern (2000).

depreciate. The presence of two stock variables in the knowledge production function mean that the accumulation of R will interact with accumulation of A.

XI. Conclusion

This paper developed a model of knowledge and R&D accumulation dynamics. The model includes two stocks, a knowledge stock representing the sum total of technologically relevant ideas, and a separate R&D stock, representing the accumulated effort devoted to the discovery of new knowledge. Prior research on economics of innovation tended to spotlight only one of these two variables, often conflating knowledge and R&D.

Our model is distinct from functional forms commonly appearing in research on rates of return to R&D—which have considered R&D stocks but apart from knowledge accumulation. It is also different from the Romer-Jones knowledge production function commonly relied on in Endogenous Growth Theory, which includes only current period discovery effort from labor employed in the R&D sector. The model takes into account the entirety of effective R&D stock into R&D-based models of growth.

Taking both R&D and knowledge stocks into account brings into the field of vision aspects of growth and innovation that theory previously left out of sight. It brings into view the possibility of ideas-driven growth that relies neither on Romer's razor-edge restriction nor on Jones' requirement of positive R&D population growth. Finally, the model paves the way for estimation of innovation and growth processes while avoiding estimation biases that likely impacted prior empirical work on productivity.

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Appendix

Griliches knowledge production and its relationship to the Perpetual Inventory Method

The stock of technologically relevant knowledge $\mathbf{A}$ is given by the following equation:

$$\mathbf{A} = G(W(B)R). \tag{A1}$$

Here, $W(B)$ represents a lag polynomial, in which B is the backward shift operator, so that:

$$W(B)R = w_0 R_t + w_1 R_{t-1} + w_2 R_{t-2} + ... = \sum_{i=-\infty}^{t} w_{t-i} R_i. \tag{A2}$$

If the constant R&D depreciation rate is γ, the lag polynomial is a geometric series with common ratio $(1-\gamma)$. Knowledge stock at time t can then be expressed as:

$$\mathbf{A}_t = \sum_{i=-\infty}^{t} (1-\gamma)^{(t-i)} R_i. \tag{A3}$$

Note that equation (A3) is a modification of equation (3) that takes into account the depreciation of stocks over time.

The current-period investment increment is not adjusted for depreciation—the addition to the stock in period t equals R_t. Previous investments, however, are adjusted for depreciation. Extracting R_t from the right-hand side of equation (A3), the total stock at time t can be decomposed into the sum of R_t and the depreciated stock from the previous period $t-1$, leading to the perpetual inventory method (PIM) equation for calculating stocks:

$$\mathbf{A}_t = R_t + (1-\gamma)\mathbf{A}_{t-1}. \tag{A4}$$

Table A1—Share in R&D Expenditure

Country	Period	Capital[a]	Labor[b]	Other[c]	Total
Argentina	1998-2011	0.09	0.71	0.20	1.00
Australia	1981-2008*	0.10	0.47	0.44	1.00
Austria	1981-2011*	0.10	0.51	0.39	1.00
Belgium	2000-2011	0.09	0.60	0.31	1.00
Chile	2007-2010	0.25	0.46	0.21	1.00
China	1998-2012*	0.19	0.24	0.56	1.00
Chinese Taipei	1998-2012	0.11	0.48	0.41	1.00
Czech Republic	1995-2012	0.14	0.38	0.48	1.00
Denmark	1981-2011*	0.09	0.56	0.35	1.00
Estonia	2005-2011	0.25	0.43	0.31	1.00
Finland	1981-2011*	0.06	0.52	0.42	1.00
France	2002-2011	0.10	0.58	0.32	1.00
Germany	1981-2011*	0.10	0.58	0.32	1.00
Greece	1995-2005*	0.15	0.59	0.26	1.00
Hungary	1992-2011*	0.13	0.43	0.39	1.00
Iceland	1981-2011*	0.10	0.58	0.32	1.00
Ireland	1981-1993	0.15	0.53	0.32	1.00
Israel	1993-2012	0.07	0.74	0.19	1.00
Italy	1981-2011*	0.12	0.56	0.32	1.00
Japan	1981-2011	0.13	0.43	0.45	1.00
Korea	1995-2011	0.14	0.38	0.47	1.00
Mexico	1993-2007*	0.19	0.56	0.24	1.00
Netherlands	1981-2011*	0.10	0.57	0.33	1.00
New Zealand	2005-2011*	0.10	0.52	0.38	1.00
Norway	1981-2011*	0.09	0.56	0.35	1.00
Poland	1994-2011	0.21	0.41	0.38	1.00
Portugal	1982-2011	0.18	0.58	0.24	1.00
Romania	1995-2011	0.12	0.49	0.39	1.00
Russian Federation	1994-2012	0.05	0.53	0.42	1.00
Singapore	1998-2012	0.20	0.42	0.38	1.00
Slovak Republic	1996-2012	0.12	0.44	0.44	1.00
Slovenia	1993-2011	0.11	0.55	0.34	1.00
South Africa	2001-2010*	0.12	0.45	0.43	1.00
Spain	1999-2011	0.17	0.56	0.27	1.00
Sweden	2007-2011*	0.04	0.40	0.32	1.00
Switzerland	1992-2008*	0.07	0.57	0.35	1.00
Turkey	2001-2011	0.17	0.47	0.36	1.00

Note: Table provides a country comparison of the cost structure of total intramural R&D for the time period indicated in the second column. Total intramural R&D includes R&D spending by government, business enterprises, higher education and private non-profit entities.
Asterisk (*) indicates that data was not available for some years during the period indicated.
[a] Consists of expenditure on equipment and buildings. [b] Expenditure on salaries. [c] Other current costs.
Source: OECD STAN Database